

Determining weak phase γ and probing new physics in $b \rightarrow s$ transitions from $B \rightarrow \eta^{(\prime)} K$

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Abstract. We present a method of determining the weak phase γ in the Cabibbo–Kobayashi–Maskawa matrix from the decays $B \rightarrow \eta K, \eta' K$ alone. Given a large ratio between color-suppressed and color-allowed tree diagrams extracted from global $\pi\pi(K)$ fits, γ is determined from the current data of $\eta' K$ and the result is in agreement with the global standard model (SM) fits. However, a smaller ratio from factorization based calculations gives $\gamma \sim 90^\circ$. New physics beyond the SM can be distinguished if γ obtained in the $\eta' K$ modes is significantly different from the ones from other modes or other approaches. The effective value of γ from $\eta' K$ is very sensitive to new physics contributions and can be used to extract new physics parameters for a class of models which do not give contributions to strong phases significantly.

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1 Introduction

Precisely obtaining the weak phase α , β and γ in the Cabibbo–Kobayashi–Maskawa (CKM) matrix is one of the central issues in the current studies of B decays. Besides global fits to all the indirect measurements in the standard model (SM) [1, 2] or measurements on the time-dependent CP asymmetry in $B \rightarrow J/\psi K_S$, the phase angles α and γ of the unitarity triangle can also be probed from hadronic charmless B decays. In the charmless decay modes $B \rightarrow PP$ with P denoting a pseudo-scalar final state, the weak phase γ can be determined either with theoretical inputs such as QCD factorization [3, 4], perturbation QCD [5–7] and soft-collinear effective theories [8–10] etc., or through model independent phenomenological methods based on flavor $SU(3)$ symmetry [11–18].

Within the flavor $SU(3)$ symmetry, direct B decay amplitudes are described by a set of flavor topological diagrams. The leading diagrams involve a tree diagram \mathcal{T} , a color-suppressed tree diagram \mathcal{C} , a flavor octet (singlet) QCD penguin diagram $\mathcal{P}(\mathcal{S})$ and a color-allowed (color-suppressed) electroweak penguin diagram \mathcal{P}_{EW} ($\mathcal{P}_{EW}C$) etc. The hierarchical structure in the size of these diagrams simplifies the analysis and makes it powerful in exploring the hadronic B decays. Recent global fits using the diagrammatic method have already shown that the weak phase γ can be determined with a reasonable precision and the obtained value agrees well with the one from the global CKM fit [19–25].

However, the current data also exhibit some puzzling patterns which need further understanding. The unexpected large branching ratio of $\pi^0\pi^0$ and the relative suppression of $\pi^+\pi^-$ are a big theoretical challenge and may require large nonfactorizable contributions [26–28]; the relative enhancement of $\pi^0\bar{K}^0$ to π^+K^- may lead to an enhancement of electroweak penguin which could be a signal of new physics (see, e.g. [24–30]). The recently measured mixing-induced CP asymmetries of $(\omega, \phi, \pi^0, \eta')K_S$, though not conclusive yet, suggest a possibility that the weak phase β obtained from $b \rightarrow s$ penguin-dominant processes may deviate from the one determined from the $b \rightarrow c$ tree-dominated process $J/\psi K_S$ [31–33].

The global fit to all the charmless B decay modes connected by flavor $SU(3)$ symmetry is the most consistent way to explore the weak phases and the involved hadronic decay amplitudes. However, to get more insight in the potential inconsistencies in the theory and a better understanding of the strong dynamics in hadronic B decays it is useful to divide the whole set of decay modes into several subsets in which the relevant parameters can be investigated individually. The comparison among the same quantities obtained from different subsets will not only provide us important cross-checks but also shed light on the origins of those puzzles and possible signals of new physics beyond the SM.

For instance, in the $\pi\pi$ system the three decays modes $\pi^+\pi^-$, $\pi^0\pi^0$ and $\pi^0\pi^-$ provide at most seven independent observables including three branching ratios, two direct CP asymmetries (the direct CP asymmetry for $\pi^0\pi^-$ is predicted to be vanishing in SM) and two mixing-induced CP asymmetries, enough to determine the in-

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involved hadronic amplitudes \mathcal{T} , \mathcal{C} , \mathcal{P} and also the weak phase γ . In $\pi\pi$ modes, the electroweak penguins are small and negligible. The recent fits taking the weak phase β as input show a good determination of all the amplitudes. The ratio of \mathcal{C}/\mathcal{T} is found to be large: close to 0.8 [23–25, 34–36], and the weak phase γ is determined up to a multi-fold ambiguity and one of them agrees well with the SM global fit value $\sim 62^\circ$. In the πK system, the available data involve four CP averaged branching ratios, three direct CP asymmetries (the direct CP asymmetry in $B^- \rightarrow \pi^- \bar{K}^0$ is predicted to be nearly zero when the annihilation diagram is negligible). Plusing a mixing-induced CP asymmetry in $B \rightarrow \pi^0 K_S$, there are eight data points in total. The independent flavor diagrams include \mathcal{T} , \mathcal{C} and \mathcal{P} . The electro-weak penguin \mathcal{P}_{EW} is significant but can be related to tree type diagrams in the SM [37, 38]. Other parameters in the CKM matrix elements can be chosen as the angles γ and β or the Wolfenstein parameter ρ and η . Thus the shape of the whole unitarity triangle can in principle be determined in the πK modes alone [39]. The current data of πK are not enough to perform such an independent determination. Taking the SM value of the weak phase γ and β as inputs, one can extract other hadronic amplitudes. The recent fits show an even larger value of $\mathcal{C}/\mathcal{T} \sim 1.7$ and enhancement of \mathcal{P}_{EW} [25, 40].

In the present paper, we discuss the determination of γ from an other important subset, the $\eta^{(\prime)}K$ modes. The advantages of using $\eta^{(\prime)}K$ final states over the $\pi\pi$ and πK states are as follows.

- All the four $\eta^{(\prime)}K$ modes are penguin dominant with appreciable tree-penguin interferences. Nonvanishing direct CP asymmetries are expected in all the four decay modes, while in the $\pi\pi$ (πK) mode one of the direct CP asymmetries in $\pi^- \pi^0$ ($\pi^- \bar{K}^0$) is predicted to be nearly zero.
- The two neutral modes in $\eta^{(\prime)}K$ will provide two additional data points from mixing-induced CP asymmetries in $\eta^{(\prime)}K_S$, while in πK there is only one.
- Most importantly, the flavor topological structure in the $\eta^{(\prime)}K$ amplitudes allows for a regrouping of penguin type diagrams in such a way that the number of independent hadronic amplitudes can be reduced to four complex parameters.
- The electroweak penguin diagram \mathcal{P}_{EW} can be included in the reduced hadronic parameters. It is not necessary to assume the SM relation between electroweak penguin and tree type diagram. This is of particular importance, as the current data imply the possibility of new physics beyond the SM.

Thus in the $\eta^{(\prime)}K$ modes there will be at most ten observables available, enough to simultaneously determine all the involved diagrammatic amplitudes, the weak phase γ and β , which determine the apex of the unitarity triangle. This method distinguishes itself from the previous ones in that it makes use of the $\eta^{(\prime)}K$ modes alone while the previous methods focus on constructing quadrangles connecting to the πK modes using $SU(3)$ symmetry [16, 41].

This paper is organized as follows. In Sect. 2, we present details of determining weak phase γ from the $\eta^{(\prime)}K$ modes.

In Sect. 3, the implications from the current data of $\eta'K$ are discussed. We take typical values of hadronic parameters as inputs to constrain γ from the $\eta'K$ modes. In Sect. 4, the new physics effects on the γ determination are discussed. We finally conclude in Sect. 5.

2 Determining γ from $\eta^{(\prime)}K$

We assume flavor $SU(3)$ symmetry and take the following diagrammatic decomposition for the $B \rightarrow \eta^{(\prime)}K$ decay amplitudes [42]:

$$\begin{aligned}\bar{\mathcal{A}}(\eta\bar{K}^0) &= \frac{1}{\sqrt{3}}(\mathcal{C} + \mathcal{P}_\eta), \\ \bar{\mathcal{A}}(\eta'\bar{K}^0) &= \frac{1}{\sqrt{6}}(\mathcal{C} + \mathcal{P}_{\eta'}), \\ \bar{\mathcal{A}}(\eta K^-) &= \frac{1}{\sqrt{3}}(\mathcal{T} + \mathcal{C} + \mathcal{P}_\eta), \\ \bar{\mathcal{A}}(\eta' K^-) &= \frac{1}{\sqrt{6}}(\mathcal{T} + \mathcal{C} + \mathcal{P}_{\eta'}),\end{aligned}\quad (1)$$

which corresponds to the flavor contents of $\eta = (-s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{3}$ and $\eta' = (2s\bar{s} + u\bar{u} + d\bar{d})/\sqrt{6}$ respectively. This is in accordance with an η_8 - η_0 mixing angle of $\theta = \arcsin(-1/3) \simeq -19.5^\circ$ [42]. Such a simple mixing scheme is a good approximation in the phenomenology and is extensively used in the recent analyses of hadronic B and D decays [43–46]. The two penguin type diagram are given by

$$\begin{aligned}\mathcal{P}_\eta &\equiv \mathcal{S} + \frac{2}{3}\mathcal{P}_{EW}, \\ \mathcal{P}_{\eta'} &\equiv 3\mathcal{P} + 4\mathcal{S} - \frac{1}{3}\mathcal{P}_{EW}.\end{aligned}\quad (2)$$

In the above expressions we assume that the color-suppressed electro-weak penguin \mathcal{P}_{EW}^C and annihilation diagram \mathcal{A} are small and negligible. We shall also assume the t -quark dominance in the penguin diagrams. With these assumptions, all the decay amplitudes depend on the four complex parameters \mathcal{C} , \mathcal{T} , \mathcal{P}_η and $\mathcal{P}_{\eta'}$. amplitudes with the definitions of The two weak phases γ and β enter the expressions from direct and mixing-induced CP asymmetries as additional free parameters. Removing an overall strong phase, there are nine real free parameters to be determined by ten observables in $B \rightarrow \eta^{(\prime)}K$ modes which include four CP averaged decay rates, four direct CP asymmetries and two mixing-induced CP asymmetries in $\eta^{(\prime)}K_S$. Although the expressions of \mathcal{P}_η and $\mathcal{P}_{\eta'}$ depend on the η - η' mixing scheme, the isospin symmetry guarantees that the neutral ($\eta(\eta')\bar{K}^0$) and charged ($\eta(\eta')K^-$) modes have the same coefficients for \mathcal{S} , \mathcal{P} and also \mathcal{P}_{EW} , which allows for the reduction to a single penguin type parameter. Thus the number of free parameters is the same for other mixing schemes such as FKS and two-mixing angle schemes (see e.g. [47, 48]).

The CP averaged branching ratio is defined through

$$\text{Br} \equiv \frac{1}{2}\tau(|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2),\quad (3)$$

where the factor τ stands for the life time differences in the B mesons and is normalized to $\tau = 1(\tau_+/ \tau_0)$ for neutral (charged) modes with $\tau_0(\tau_+)$ being the life time for neutral (charged) B mesons and $\tau_+/\tau_0 = 1.086$. The definition of direct CP asymmetry is

$$a_{CP} \equiv \frac{|\bar{\mathcal{A}}|^2 - |\mathcal{A}|^2}{|\bar{\mathcal{A}}|^2 + |\mathcal{A}|^2}. \quad (4)$$

The mixing-induced CP asymmetry is defined as

$$a_{CP}(t) \equiv \frac{\text{Br}(\bar{B}^0(t) \rightarrow f) - \text{Br}(B^0(t) \rightarrow f)}{\text{Br}(\bar{B}^0(t) \rightarrow f) + \text{Br}(B^0(t) \rightarrow f)} = S \sin(\Delta m_B t) - C \cos(\Delta m_B t), \quad (5)$$

where

$$S = \frac{\text{Im}\lambda}{|\lambda|^2 + 1}, \text{ and } \lambda = -e^{-2i\phi_d} \frac{\bar{\mathcal{A}}}{\mathcal{A}}, \quad (6)$$

with ϕ_d the weak phase appearing in $B^0-\bar{B}^0$ mixing and $\phi_d = \beta$ in the SM. The coefficient C is related to the direct CP asymmetry by $C = -a_{CP}$. The latest data involving $B \rightarrow \eta^{(\prime)}K$ are summarized in Table 1 [49–51].

It is well known that the unusually large branching ratios of $B \rightarrow \eta'K$ modes may require an enhancement of the flavor singlet penguin diagrams \mathcal{S} , which present another theoretical challenge and is still under extensive theoretical study (see e.g. [52]). The flavor singlet contribution can be systematically calculated in QCD factorization, and the results favor a smaller value with significant theoretical uncertainties [53]. However, for the purpose of extracting the weak phases one needs only the ratios of the decay rates between neutral and charged modes in which the penguin amplitudes cancel to a great extent, making the results insensitive to \mathcal{S} . We then define a ratio between the neutral and charged decay rates by

$$R^{(\prime)} \equiv \frac{\tau_+}{\tau_0} \frac{\text{Br}(\eta^{(\prime)}\bar{K}^0)}{\text{Br}(\eta^{(\prime)}K^-)}. \quad (7)$$

The current data of $B \rightarrow \eta'K$ give

$$R' = 1.04 \pm 0.08. \quad (8)$$

The corresponding ratio in the ηK modes gives $R < 0.83$. The ratio between tree and penguin type diagrams are parameterized as

$$\zeta^{(\prime)} e^{i\delta^{(\prime)}} = \frac{\mathcal{C}}{\mathcal{P}_{\eta^{(\prime)}}} e^{i\gamma}, \quad \chi^{(\prime)} e^{i\omega^{(\prime)}} = \frac{\mathcal{T} + \mathcal{C}}{\mathcal{P}_{\eta^{(\prime)}}} e^{i\gamma}, \quad (9)$$

where $\zeta^{(\prime)}$ and $\chi^{(\prime)}$ are both real-valued. $\delta^{(\prime)}$ and $\omega^{(\prime)}$ are purely strong phases as the weak phase γ has been extracted from the definitions. We further define the ratio between color-suppressed and color-allowed tree diagrams:

$$r e^{i\varphi} \equiv \frac{\zeta^{(\prime)} e^{i\delta^{(\prime)}}}{\chi^{(\prime)} e^{i\omega^{(\prime)}}} = \frac{\mathcal{C}}{\mathcal{T} + \mathcal{C}}, \quad (10)$$

with $r = |\zeta^{(\prime)}/\chi^{(\prime)}|$ and $\varphi = \delta^{(\prime)} - \omega^{(\prime)}$, which are common to both the ηK and the $\eta'K$ modes.

All the parameters can be solved numerically from the above equations. They can also be solved analytically to the leading order expansion of $\zeta^{(\prime)}$ and $\chi^{(\prime)}$. Taking the $\eta'K$ modes as an example, to the leading order of ζ' and ω' , the ratio of the decay rates is given by

$$R' \equiv 1 + \Delta R' \simeq 1 + 2\zeta' \left[\cos \delta' - \frac{1}{r} \cos(\delta' - \varphi) \right] \cos \gamma. \quad (11)$$

The two direct CP asymmetries are

$$a'_0 \equiv a_{CP}(\eta'\bar{K}^0) \simeq 2\zeta' \sin \delta' \sin \gamma, \quad (12)$$

$$a'_- \equiv a_{CP}(\eta'K^-) \simeq 2\frac{\zeta'}{r} \sin(\delta' - \varphi) \sin \gamma. \quad (13)$$

The mixing-induced CP violation is found to be

$$S' \equiv S(\eta'K_S) \simeq \sin 2\beta + 2\zeta' \cos 2\beta \cos \delta' \sin \gamma. \quad (14)$$

In the above expressions, we use the primed quantities such as R' , a'_0 , a'_- and S' to denote the ratio of decay rates, and direct and mixing-induced CP asymmetries, respectively, in the $\eta'K$ modes. For the $B \rightarrow \eta K$ process, equations similar to (11)–(14) can be constructed with the substitution of primed quantities for unprimed ones, i.e. R , a_0 , a_- and S

Table 1. The latest world average of the branching ratios (in units of 10^{-6}), direct CP violations as well as mixing-induced CP violations for the $B \rightarrow \eta K, \eta'K$ modes. The direct CP asymmetry of $\eta'\bar{K}^0$ comes from time-dependent CP asymmetry measurements of $\eta'K_S$

	CLEO	BaBar	Belle	WA
$\text{Br}(\eta\bar{K}^0)$	< 9.3	< 2.5	< 2.0	< 2.0
$\text{Br}(\eta K^-)$	$2.2^{+2.8}_{-2.2}$	$3.3 \pm 0.6 \pm 0.3$	$2.1 \pm 0.6 \pm 0.2$	2.6 ± 0.5
$\text{Br}(\eta'\bar{K}^0)$	$89^{+18}_{-16} \pm 9$	$67.4 \pm 3.3 \pm 3.3$	$68 \pm 10^{+9}_{-8}$	68.6 ± 4.2
$\text{Br}(\eta'K^-)$	$80^{+10}_{-9} \pm 7$	$68.9 \pm 2 \pm 3.2$	$78 \pm 6 \pm 9$	70.8 ± 3.4
$a_{CP}(\eta K^-)$		$-0.2 \pm 0.15 \pm 0.01$	$-0.49 \pm 0.31 \pm 0.07$	-0.25 ± 0.14
$a_{CP}(\eta'\bar{K}^0)$				(0.04 ± 0.08)
$a_{CP}(\eta'K^-)$	$0.03 \pm 0.12 \pm 0.02$	$0.033 \pm 0.028 \pm 0.005$	$-0.015 \pm 0.007 \pm 0.009$	0.027 ± 0.025
S'		$0.30 \pm 0.14 \pm 0.02$	$0.65 \pm 0.18 \pm 0.04$	0.43 ± 0.11

etc. Equations (11)–(14) together with the ones for the ηK modes provide eight equations which constrain the eight parameters,

$$\zeta', \delta', \zeta, \delta, r, \varphi, \gamma \text{ and } \beta. \quad (15)$$

A simultaneous determination of γ and β will allow for a reconstruction of the unitarity triangle from the $\eta^{(\prime)}K$ modes alone.

Since great success has already been achieved in the measurement of $\sin 2\beta$ from $B \rightarrow J/\psi K_S$ in the two B -factories and the value obtained agrees remarkably with the one from global fits to all the indirect measurements such as neutral B and K meson mixing and semileptonic B decays etc., throughout this paper we shall take the value of [54, 55]

$$\sin 2\beta = 0.687 \pm 0.032 \quad (16)$$

from $B \rightarrow J/\psi K_S$ as input and focus on the determination of the less known weak phase γ in the $\eta^{(\prime)}K$ modes.

Following this strategy, the values of r and φ are determined purely by direct CP asymmetries and β :

$$\tan \varphi \simeq \frac{(a'_0 a_- - a_0 a'_-) \cos 2\beta}{a_-(S' - \sin 2\beta) - a'_-(S - \sin 2\beta)}, \quad (17)$$

and

$$r \simeq \frac{a'_0}{a'_-} \left(\cos \varphi - \frac{S' - \sin 2\beta}{a'_0 \cos 2\beta} \sin \varphi \right). \quad (18)$$

Note that the determination of φ requires CP asymmetry measurements for both the ηK and the $\eta' K$ modes. The solution to γ in terms of r , φ and β is found straightforwardly:

$$\tan \gamma \simeq \frac{1}{r \Delta R'} \left[(r - \cos \varphi) \frac{S' - \sin 2\beta}{\cos 2\beta} - a'_0 \cdot \sin \varphi \right]. \quad (19)$$

Thus γ is determined up to discrete ambiguities. The above expression forms the base of the present paper. The weak phase γ does not depend on the ratio ζ' and χ' . It only depends on the ratio between the tree type diagrams r and φ . The accuracy of γ depends heavily on the CP violation measurements. It also depends on the ratios of the decay rates R' . Note again that in this method the weak phase γ is determined within a closed subset of $\eta^{(\prime)}K$. No measurements from other modes are needed. In the typical case where φ is small and $r < \cos \varphi$, the second term in the right handed side of (19) is negligible, and the sign of $\tan \gamma$ depends on the sign of $(S' - \sin 2\beta)/\Delta R'$. Thus a positive $\tan \gamma$ nontrivially requires $R' > 1$ since the current data prefer $S' - \sin 2\beta < 0$. The value of $\tan \gamma$ will be enhanced if r or $\Delta R'$ is very small.

The main source of the uncertainties comes from the SU(3) breaking between the ηK and $\eta' K$ decay amplitudes. At present there are no robust estimates for SU(3) breaking effects. In the naive factorization approach the SU(3) breaking arises from two different pieces in the

$B \rightarrow \eta^{(\prime)}K$ amplitudes. One is proportional to the form factors $F_0^{B \rightarrow \eta}(m_B^2 - m_\eta^2)/F_0^{B \rightarrow \eta'}(m_B^2 - m_{\eta'}^2) \approx 1.16$, with $F_0^{B \rightarrow \eta^{(\prime)}}$ the form factor of the $B \rightarrow \eta^{(\prime)}$ transition. The other one is proportional to the decay constants $f_\eta^u/f_{\eta'}^u \approx 1.22$. This gives an estimate that the SU(3) breaking effect is up to $\sim 20\%$. It needs to be emphasized that P_η and $P_{\eta'}$ are treated as two independent parameters not related by SU(3) symmetry. Note that the SU(3) symmetry could be broken in a more complicated way in the strong phase [56] and radiative corrections may give contributions not proportional to the decay constants [53]. The accuracy of r , φ and $\delta^{(\prime)}$ depends on the precision of the a_{CP} to be measured from the $\eta^{(\prime)}K$ modes. The branching ratios for $\eta' K$ are known to be large (a few $\times 10^{-5}$), while the ηK modes are expected to be an order of magnitude smaller due to its flavor structure [57]. However, in the ηK modes the tree–penguin interferences could be stronger and the direct CP asymmetries could be more significant. With the increasing statistics in the two B -factories, the precision of $a_{CP}(\eta' K)$ will be improved. Higher precision measurements can be achieved in the future super- B factories [58].

3 Implications from the latest data

The weak phase γ obtained from the $\eta^{(\prime)}K$ modes can be compared with the one from other methods. The difference, if it exists, will shed light on the nonstandard contributions or possible new physics. At present, the data of the direct and mixing-induced CP asymmetries for $\eta \bar{K}^0$ are not yet available, and one cannot have a practical estimate of γ from (19). However, r and φ can be extracted from other modes or calculated theoretically. Taking r and φ as inputs, one can infer the value of γ from the $\eta' K$ modes using the current data and compare it with the SM fit value. For illustrations, we consider two typical sets for the value of r and φ .

(A) The values of r and φ are extracted from global $\pi\pi$ and πK fits based on flavor SU(3) symmetry. All the recent fits prefer a large \mathcal{C} [24, 25, 34–36, 59]. From an up-to-date fit in [25], one finds the following values:

$$r = 0.56 \pm 0.05, \quad \varphi = (-33.2 \pm 6.3)^\circ. \quad (20)$$

The large r is driven by the observed large branching ratio of $\pi^0 \pi^0$. The value of r obtained in the $\pi\pi$ and πK fits can be directly used in $\eta' K$ as the leading SU(3) breaking effects cancel in the ratio between \mathcal{C} and \mathcal{T} .

(B) The values of r and φ are taken from the QCD factorization calculations [4, 60], which prefer smaller values with considerable uncertainties. In numerical estimations we take the following typical values from the latest QCD factorization estimate [60]:

$$r = 0.20 \pm 0.14, \quad \varphi = -(12 \pm 18)^\circ. \quad (21)$$

In Fig. 1 we plot γ as a function of S' , taking (20) as inputs for the three different values of $R' = 0.96, 1.04$ and

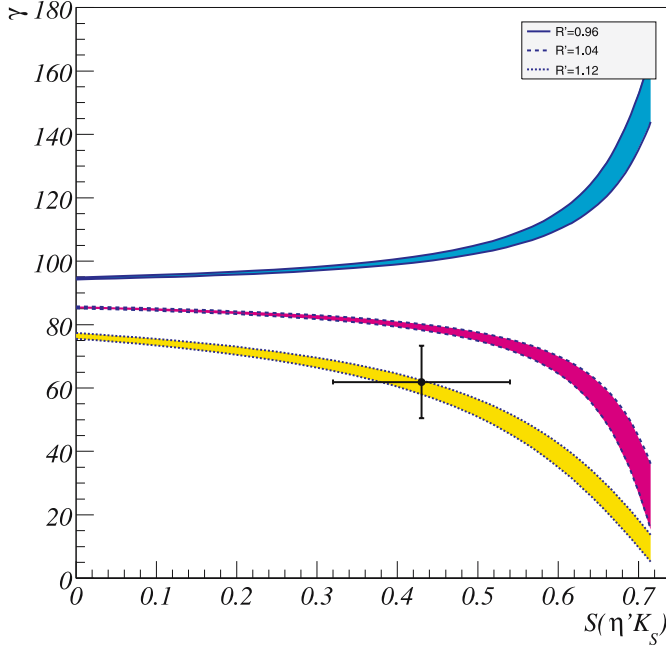


Fig. 1. Weak phase γ (in degrees) as a function of S' . The solid, dashed and dotted bands correspond to $R' = 0.96, 1.04$, and 1.12 respectively. The cross indicates the current experimental measurements with the horizontal bar representing the data of S' and the vertical one representing the favored range of γ from a global SM fit. The values of r and φ are taken from (20) with uncertainties taken into account

1.12 respectively, corresponding to the 1σ allowed range. The figure shows a strong dependence of γ on both S' and R' . For $R' < 1$, γ grows with S' increasing and is always larger than the best fitted value from the global CKM fit. For $R' > 1$, it moves down to the opposite direction and reaches $\sim 60^\circ$ for $S' \simeq 0.5$. For $R' = 1$, $\tan \gamma$ becomes infinity which fixes γ at $\sim 90^\circ$. The current data of R' cannot definitely tell us if R' is greater or smaller than unity. To have a robust conclusion, higher precision data are urgently needed.

From Fig. 1, one finds a overall consistency with the global SM fit. For S' and R' varying in the 1σ range, the value of γ is found to be

$$45^\circ \lesssim \gamma \lesssim 110^\circ. \quad (22)$$

The error is still significant and the center value gives a slightly large $\gamma \sim 78^\circ$.

Note that some previous analyses found problems with a small S' [23, 61]. The difference mainly originates from the data used in the fits. In the present paper, we use the updated data while in the previous ones the old data of $\text{Br}(\eta' K^-) = 77.6 \pm 4.6$ and $\text{Br}(\eta' \bar{K}^0) = 65.2 \pm 6.2$ are used which correspond to $R' \simeq 0.91$. As it is shown in Fig. 1, a small $R' < 1$ will not make a good fit.

In Fig. 2, a similar plot is made with the values of r and φ taken from (21). Comparing with Fig. 1, one sees a smoother dependence on R' and S' , and for a smaller $r = 0.20$ from (21) and R', S' in the 1σ range, the value of

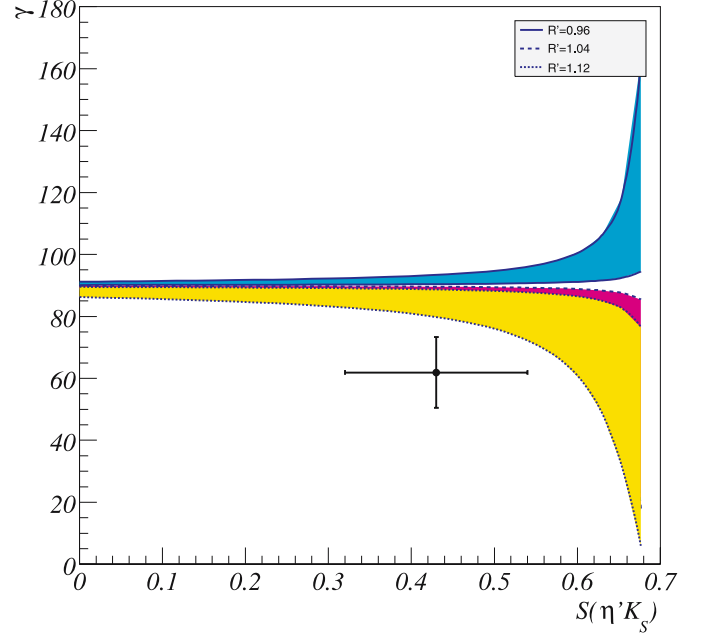


Fig. 2. The same as Fig. 1 while the values of r and φ are taken from (21)

γ is found to be confined in a narrow range of

$$85^\circ \lesssim \gamma \lesssim 95^\circ. \quad (23)$$

Clearly, a large $\gamma \sim 90^\circ$ is favored in this case. The reason is that the smaller r enhances $\tan \gamma$, making the three curves closing in to each other and forcing γ to be $\sim 90^\circ$. In this case γ can reach $\sim 60^\circ$ only for $S' = 0.6 \sim 0.7$, i.e. close to the $\sin 2\beta$ from $B \rightarrow J/\psi K_S$. One has to bear in mind that the measurement on S' is not very conclusive yet as there still exists a discrepancy between Babar and Belle results [49, 50]. Using the PDG average method, the error should be enlarged by a factor of $\sqrt{\chi^2}$ which is the square root of the chi-square value of the average. This gives $S' = 0.43 \pm 0.17$. However, a large γ is still favored in the enlarged region. The theoretical prediction to S' based on QCD factorization prefers that S' is slightly greater than $\sin 2\beta$, $S' - \sin 2\beta \simeq 0.01$ [31, 32]. This remains to be tested in a future experiment.

It follows from the above results that if γ is indeed around 62° , a large r is favored by the current data of $\eta' K$ only, which is independent of the data of $\pi\pi$ and πK . Independent determination of the relative size of the color-suppressed tree diagram may provide us with important hints on its origin. A possible explanation is that the extracted \mathcal{C} is an effective amplitude involving other important contributions such as a large nonfactorizable W -exchanging diagram \mathcal{E} [26–28], a large penguin type diagram contribution through internal $c\bar{c}$ loops, i.e. the charmed penguin [62, 63], large final state interactions [64, 65] etc. The exchange diagram \mathcal{E} only contributes to the $\pi\pi$ modes and will not affect πK and $\eta' K$. The charmed penguin always comes together with the ordinary penguin diagrams. But the tree-penguin interferences are

different in $\pi\pi$, πK and $\eta'K$. One cannot expect a universal enhancement pattern of \mathcal{C} in all modes. The final state interaction is more process-dependent. Thus if the ratio r can be precisely determined independently from various subsets, it is possible to distinguish some of the explanations. For instance, if a large r is confirmed in all the $\pi\pi$, πK and $\eta^{(\prime)}K$ modes, the first explanation will not be favored.

4 New physics effects

We proceed to discuss the new physics contributions. When the weak phase β is taken as known from $B \rightarrow J/\psi K_S$, there are eight data points to constrain seven real parameters in the $\eta^{(\prime)}K$ system. The nonzero degree-of-freedom allows one to make cross-checks for consistency or explore new physics contributions.

The new physics may affect the observables in two different ways. One is through modifying $B^0-\bar{B}^0$ mixing which makes $\phi_d \neq \beta$. The consequence is that the mixing induced CP asymmetry for all the modes will be affected in the same manner, which is not very likely as the measurement of $\sin 2\beta$ from $J/\psi K_S$ agrees remarkably with all the indirect measurements of the unitarity triangle, and so far no systematic deviations of $\sin 2\beta$ from its global SM fit value are confirmed in other modes. The other way is that new physics contributes to the decay amplitudes, most likely through $b \rightarrow s$ loop processes. In this case the modifications to direct and mixing-induced CP asymmetries will be process dependent.

Taking the $\eta'K$ modes as an example, we parameterize the new physics contribution to $b \rightarrow s$ penguin in the following form:

$$\begin{aligned}\bar{A}(\eta' \bar{K}^0) &= \frac{1}{\sqrt{6}} \mathcal{P}_{\eta'} \left[1 + \zeta' e^{i\delta'} + \xi_s e^{i(\delta_s + \phi_s)} \right], \\ \bar{A}(\eta' K^-) &= \frac{1}{\sqrt{6}} \mathcal{P}_{\eta'} \left[1 + \chi' e^{i(\delta' - \varphi)} + \xi_s e^{i(\delta_s + \phi_s)} \right],\end{aligned}\quad (24)$$

where δ_s and ϕ_s are the strong and weak phases generated by new physics. Its size relative to $\mathcal{P}_{\eta'}$ is denoted by ξ_s . For simplicity, we assume that the new physics contribution respects the isospin symmetry under $u \leftrightarrow d$. This happens to the modes mainly contributing to the QCD penguins [66]. For any specific models such as the two-Higgs-doublet model [67–70], the Z' model [71] etc., the relation between them is computable. In the presence of new physics, the expressions for CP asymmetries to the leading order are modified as follows:

$$\begin{aligned}a'_0 &\simeq 2\zeta' \sin \delta' \sin \gamma - 2\xi_s \sin \delta_s \sin \phi_s, \\ a'_- &\simeq 2\chi' \sin(\delta' - \varphi) \sin \gamma - 2\xi_s \sin \delta_s \sin \phi_s,\end{aligned}\quad (25)$$

and the mixing-induced CP asymmetry is given by

$$S' \simeq \sin 2\beta + 2\zeta' \cos 2\beta \cos \delta' \sin \gamma - 2\xi_s \cos 2\beta \cos \delta_s \sin \phi_s.$$

Note that in this case R' is not affected as the new physics contributions to the charged and neutral modes cancel.

The difference between the two direct CP asymmetries $a'_0 - a'_-$ is not affected either. In the presence of new physics, the weak phase γ extracted from (19) will be an effective one denoted by $\tilde{\gamma}$, and is related to the true value of γ through

$$\begin{aligned}\tan \tilde{\gamma} &\equiv \frac{1}{r\Delta R'} \left[(r - \cos \varphi) \frac{S' - \sin 2\beta}{\cos 2\beta} - \sin \varphi \cdot a'_0 \right] \\ &\simeq \tan \gamma - 2 \frac{r \cos \delta_s - \cos(\delta_s - \varphi)}{r\Delta R'} \xi_s \sin \phi_s.\end{aligned}\quad (26)$$

Thus the deviation of the effective value $\tilde{\gamma}$ from the true γ is a measure of the new physics effects which can be used to extract new physics parameters or distinguish different new physics models [72]. The true value of γ can be obtained from other measurements such as through $B \rightarrow DK$ [73] or from global CKM fits. The new physics effects will be enhanced if the deviation of R' from unity is tiny. As the current data give a central value of $R' \simeq 1.04$, the effective $\tilde{\gamma}$ is very sensitive to new physics. If the true value of γ is indeed around 62° , for typical values of r and φ taken from (20) and $R' = 1.04$, the enhancement factor is about ~ 50 . As a consequence, a significant difference of a few tens degrees between $\tilde{\gamma}$ and γ is possible for $\xi_s \sin \phi_s \sim 0.1$.

It has been argued recently that in general the new physics will not generate significant relative strong phases as the strong phases mainly originate from the long-distance rescatterings of the final states while new physics contributes only to the short-distant part [74]. In the case that the new physics strong phase δ_s is negligible, the combined new physics parameter $\xi_s \sin \phi_s$ can be directly extracted. As an illustration, we take the central value of $r = 0.56$ and $\varphi = -33^\circ$ from (20) and R' in the 1σ range, which gives

$$0 \lesssim \xi_s \sin \phi_s \lesssim 0.25.\quad (27)$$

It follows that for a large r , the current data marginally agree with the SM, and the new physics receives only an upper bound.

For a smaller value of $r = 0.2$ and $\varphi = -12^\circ$ in (21), a positive signal of nonzero $\xi_s \sin \phi_s$ is found:

$$0.15 \lesssim \xi_s \sin \phi_s \lesssim 0.19,\quad (28)$$

which demonstrates that the $\eta^{(\prime)}K$ mode provides a good avenue to explore new physics contributions. Needless to say that the current experimental status is not conclusive yet, and one cannot draw a robust conclusion on the presence of new physics. The advantage of using (26) in the $\eta'K$ modes to probe new physics is that besides new physics parameters the difference between the effective and the true γ only depends on the hadronic parameters r and φ . Knowledge of the tree-penguin ratio $\zeta^{(\prime)}$ and $\chi^{(\prime)}$ is not needed. Comparing with probing new physics through $B_s \rightarrow KK$, although the flavor structure in $B_s \rightarrow KK$ is simpler, the tree-penguin interference cannot be avoided and one has to combine it with $B \rightarrow \pi\pi$ where additional assumptions on new physics effects in the $b \rightarrow d$ penguin have to be made [75].

5 Conclusion

In summary, we have presented a method for an independent determination of the weak phase γ from $B \rightarrow \eta^{(\prime)}K$ alone, which makes use of measurements of all the direct and mixing-induced CP asymmetries. The value of γ extracted from $\eta^{(\prime)}K$ may be compared with the ones from other modes. The possible discrepancy may help us to understand the current puzzles in charmless B decays. We have taken two sets of the ratio \mathcal{C}/\mathcal{T} as inputs to analyse the implications of the recent data on the $\eta^{(\prime)}K$ modes. One is from global $\pi\pi$, πK and KK fits which leads to a $45^\circ \lesssim \gamma \lesssim 110^\circ$ in agreement with the SM fit value. The other is from QCD factorization calculations which makes γ lie around 90° . Within the SM, it implies that a large \mathcal{C} is independently favored in the $\eta^{(\prime)}K$ modes. New physics beyond SM can be isolated if γ obtained in the $\eta^{(\prime)}K$ modes is significantly different from the ones from other decay modes or other approaches. The values of γ obtained from $\eta^{(\prime)}K$ are found to be sensitive to new physics contributions and can be used to extract new physics parameters if the new physics does not carry significant new strong phases.

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